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Spectral Analysis of Pulse-Modulated rf Signals

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Abstract

The parameters that characterize a rectangular-shaped pulse-modulated sinusoidal signal are the carrier frequency, the pulsewidth, the repetition frequency, and the number of pulses in or the duration of the signal. We use a Fourier series representation to show the influence of these parameters on the spectrum of a pulse-modulated signal at a microwave carrier frequency. When an additional amplitude modulation is applied at audio frequencies, the resulting transient cannot be efficiently analyzed with numerical transform techniques. We present approximate numerical and analytical techniques to obtain the frequency spectrum of such signals. This approach allows the near-real-time spectral analysis of modulated signals. Thus, the resulting spectrum can be easily calculated for idealized modulation waveforms. A typical example is presented and the effect of pulse modulation on the spectral content of an rf signal burst is discussed.

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1. Introduction

A periodic time function $f(t)$ with a period T_0 can be represented as an infinite sum of exponential functions. In particular, with an angular frequency $\omega_0 = 2\pi/T_0 = 2\pi$ (frequency) υ_0,

$$f(t) = \sum_{n=-\infty}^{n=\infty} \alpha_n e^{jn\omega_0 t} \ , \tag{1}$$

is an exponential Fourier series expansion with coefficients

$$\alpha_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} f(t) e^{-jn\omega_0 t}\, dt \ . \tag{2}$$

The Fourier series is a valid representation when the Dirichlet conditions are satisfied, which requires $f(t)$ to be a finite periodic time function.[1] The time function must be finite in the sense that it has a finite number of maxima, minima, and discontinuities in every finite interval.

For now, $f(t)$ is considered an infinite-duration pulse train,

$$f(t) = \sum_{n=-\infty}^{n=\infty} f_0\left(t + nT_0\right), \text{ where } f_0(t) = \begin{cases} f(t), & |t| \le T_0/2 \\ 0, & \text{otherwise} \end{cases} \ . \tag{3}$$

Then the Fourier integral of $f_0(t)$,

$$F_0(\omega) = \int_{-\infty}^{\infty} f_0(t) e^{-j\omega t} dt = \int_{-T_0/2}^{T_0/2} f(t) e^{-j\omega t} dt \ , \tag{4}$$

is the continuous spectrum of one period of the signal $f(t)$. A periodic rectangular pulse has an approximately band-limited spectrum, or $F_0(\omega) << F_0(0)$ for $|\omega| < \omega_{max}$, where we take $\omega_{max} = 20\pi/T_0$. The Fourier transform representation of the signal spectrum $F(\omega)$ is a sequence of impulses

$$F(\omega) = \frac{2\pi}{T_0} \sum_{n=-\infty}^{n=\infty} F_0\left(n\omega_0\right)\delta\left(\omega - n\omega_0\right) \ , \tag{5}$$

defined by the spectral envelope, $F_0(\omega)$. Then the Fourier series coefficients are $F_0(\omega)/T_0$, evaluated at $\omega_n = n\omega_0 = 2\pi n/T_0$, or[2]

$$\alpha_n = \frac{1}{T_0} F_0(\omega)\big|_{n\omega_0} \ . \tag{6}$$

The band-limited representation includes the periodic extensions of the fundamental spectrum $F_0(\omega)$ for all frequencies, which implies that $f(t)$ is infinite in duration. I relied on the band-limited approximation throughout to develop analytical and numerical PC tools to obtain the spectrum for this class of signals. I used MATLAB® to calculate the band-limited spectrum of periodic signals that modulate a single rf carrier at angular frequency $\omega_c = 2\pi f_c$. The physical rf signal is a burst of pulses and the

[1] A. Papoulis, *The Fourier Integral and Its Applications*, New York, NY: McGraw-Hill (1987).

[2] A. V. Oppenheim, A. S. Willsky, and S. H. Nawab, *Signals and Systems*, 2nd ed, Upper Saddle River, NJ: Prentice Hall (1997).

sum in equation (3) would be truncated to $2N + 1$ pulses. The pulse train is truncated with a rectangular pulse window function in the time domain (time-windowing) that corresponds to a frequency-domain convolution.[1]

Consider a repetitive rectangular pulse-modulated rf carrier, where the modulating pulse is assumed to be ideal and has negligible rise- and fall-times compared to the width T. An additional amplitude modulation (AM) with the periodic function $g(t)$ that has a period $T_2 \sim kT_0$ for integer k is often applied to the pulse-modulated rf signal. The fundamental spectrum can be obtained from sampled transient data $q_0(t_n) = f_0(t_n)g_0(t_n)$ over one period using numerical transform techniques such as the fast Fourier transform (FFT). For low-frequency modulations (i.e., $k >> 1$), this approach quickly becomes numerically intensive if the high-frequency carrier is included in the sampled transient. When all modulations have a low-frequency spectrum compared to the rf carrier, we need only sample $q_0(t)$ sufficiently to resolve the highest frequency component of the modulation waveform. In this case, the MATLAB FFT routines can be used to efficiently obtain the spectrum for the modulation waveform, which is then shifted to the single carrier frequency. I show that the rf modulation determines the spectral bandwidth (BW) of the modulated signal with impulses at the rf pulse repetition frequency (PRF). Additional modulations reduce the peak amplitude according to the AM duty factor and introduce impulses at the AM PRF. When only the spectral envelope is of interest, $Q_0(\omega)$ can be directly calculated in the frequency domain (for idealized modulation waveforms) to approximate the modulated rf spectrum. Numerical and analytical approaches are compared and used to investigate the effect of rectangular pulse modulation on the spectral content of a finite-duration rf burst.

[1]A. Papoulis, *The Fourier Integral and Its Applications.*

2. Pulse-Modulated rf Signal

A rectangular pulse $p_X(t)$ symmetric about the time reference ($t = 0$) with unit amplitude and half pulsewidth $X = T/2$ has the frequency spectrum

$$P_X(\omega) = \frac{2\sin\left(\frac{\omega T}{2}\right)}{\omega} \ , \tag{7}$$

with peak magnitude $2X = T$ and full bandwidth (FBW) $1/X = 2/T$. We use this ideal pulse to modulate a microwave carrier where $f_0(t) = p_X(t)c(t) = p_X(t)\cos(\omega_c t)$ is a repetitive pulse-modulated sinusoidal signal. The carrier spectrum is a single frequency, as represented by $C(\omega) = 1/2\{\delta(\omega + \omega_c) + \delta(\omega - \omega_c)\}$.

Then $f(t)$ is a real, even function of time with Fourier coefficients

$$\alpha_n = \frac{2}{T_0}\int_0^{T_0/2} f(t)\cos\left(n\omega_0 t\right)dt \ . \tag{8}$$

The partial sum $f_N(t) \sim f(t)$ is

$$f_N(t) = \alpha_0 + 2\sum_{n=1}^{N} \alpha_n \cos(2\pi nt/T) \ , \tag{9}$$

which can be written as an average over one period of $f(t)$ with the Fourier kernel $k_N(t)$ as a weighting function.[1] That is, $f_N(t)$ can be made to approximate $f(t)$ to an arbitrary accuracy in the interval $|t| < T_0/2$ by choosing more sinusoids in the expansion. The Fourier series coefficients for $f(t)$ are $\alpha_n = F_0(n\omega_0)/T_0$, where

$$F_0(\omega) = \frac{1}{2}\left\{P_X\left(\omega - \omega_c\right) + P_X\left(\omega + \omega_c\right)\right\} \ . \tag{10}$$

Compared to equation (9), the positive frequency component of this fundamental spectrum has the same FBW but half the magnitude. Although this band-limited spectrum implies an infinite-duration signal, there is no ambiguity in considering a finite number of samples. However, by truncating equation (1) to a finite number of terms, we also truncate the Fourier series representation of the signal spectrum in equation (5). This reduces the resolution in the frequency domain so that care must be taken to adequately sample the modulation waveforms.

2.1 Numerical Results

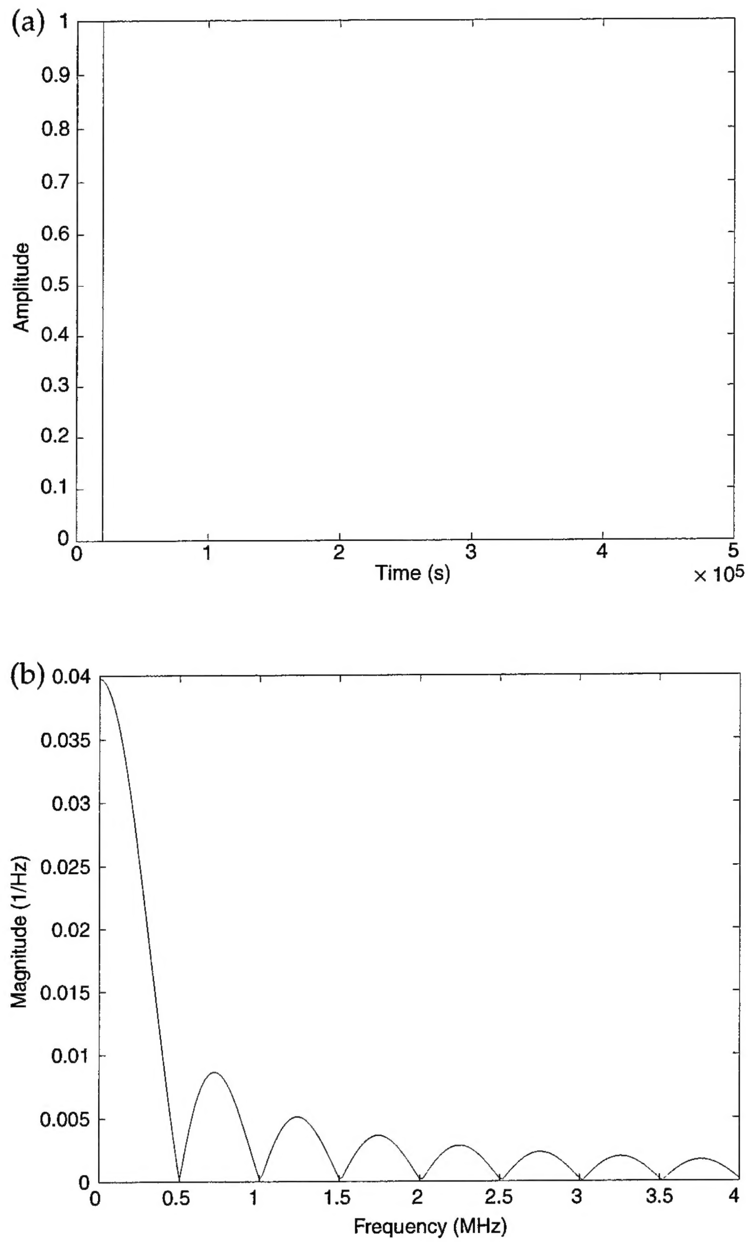
We consider a repetitive rectangular pulse modulation waveform $p_X(t)$ with constant pulsewidth (T) and PRF. The signal is periodic with period $T_0 = 1/\text{PRF}$ and rf duty factor $D_{rf} = T/T_0$ in each period of the modulation. Since the periodic extensions of the fundamental spectrum are part of the FFT, the FFT result corresponds to an infinite rectangular pulse train. For example, let the carrier frequency be 1.3 GHz with $T = 2$ µs and $T_0 = 50$ µs, assuming a unit amplitude (typically peak transmitted power)

[1] A. Papoulis, *The Fourier Integral and Its Applications.*

signal. A single period of the modulation pulse is shown in figure 1(a), where 201 time samples per pulse ($dt = T/201$) were used to define one period of the transient $p_X(t)$. The FFT of this modulation waveform $P_X(\omega)$ is the continuous spectrum shown in figure 1(b). Notice that the spectrum peak magnitude is the signal time-average (or dc-component) in the transient time window D_{rf} = 4 percent. An rf carrier modulated by this waveform $f(t)$ is represented by samples at fixed intervals dt and the FFT used to calculate $F(\omega)$. A sample size of $dt = (20f_c)^{-1}$ is recommended (i.e., 20 samples per carrier period), and the sampling error increases rapidly as dt is increased.

One period of the modulated rf signal $f_0(t)$ that includes the rf carrier is discretized for numerical analysis. The FFT result for the fundamental

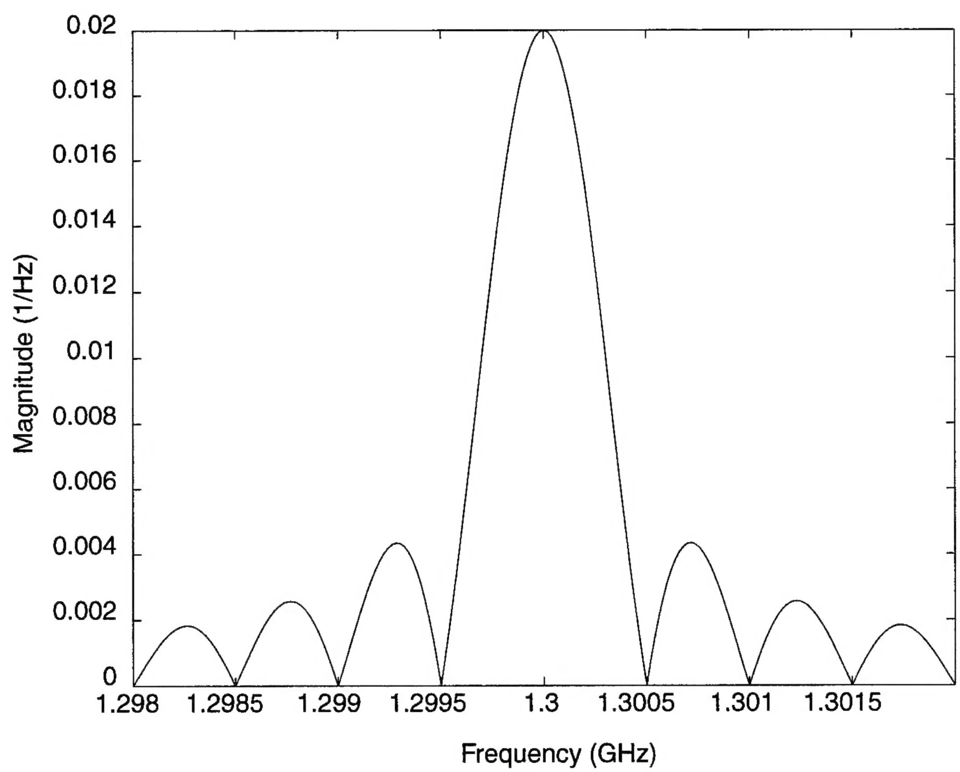
Figure 1. (a) Single period of a rectangular pulse modulation waveform and (b) single rectangular pulse FFT with 4 percent duty factor.



spectrum $F_0(\omega)$ corresponds to the spectral envelope of the infinite-duration signal $f(t)$. The spectrum is shown in figure 2 for positive frequencies using 20 time samples per period of the 1.3-GHz carrier. The negative frequency components in equation (10) cannot be ignored as reflected in the FFT result. For a modulated sinusoid, the power spectrum peak magnitude is one-half the peak power time-average in one period of the modulation waveform or one-half the rf duty factor, $\frac{1}{2}D_{rf}$ = 2 percent. For the fundamental modulation waveform (i.e., one complete period) the FFT result $F_0(\omega)$ is the continuous spectral envelope with FBW = $1/X$ = $2/T$ about the carrier f_c. Thus, the spectrum of the infinite-duration rf signal $F(\omega)$ is a sequence of impulses spaced at $n\omega_0$ with envelope $|F_0(\omega)|$ as in figure 2.

In practice, the modulation pulse shapes and carrier frequency are very stable, so that the waveform approximates the pulse train in equation (3) and modulates a single carrier frequency. The power spectrum has FBW = $2/T$ about the carrier and magnitude $\frac{1}{2}D_{rf}$ with impulses at the rf PRF. Measurement of the modulation pulse waveform along with the rf modulation parameters is sufficient to completely characterize the transmitted signal. This measurement is often a digitized version of the pulse waveform along with the modulation parameters and peak transmitted power from which the spectrum can be obtained. In our example of a modulated rf signal, the required number of time samples is 6.5×10^5, and the analysis required several minutes for the results shown in figure 2. More typical is PRF << 10 kHz, so that the required number of time samples rapidly increases to $>10^6$. Making such computations would not be a problem for modern computer resources, but we desire to analyze the

Figure 2. Single pulse-modulated 1.3-GHz carrier FFT with 2 percent rf duty factor.



spectrum for more complex modulations in near real-time. As the time window required to include one full period of the modulated signal increases, the number of time samples (with linear spacing) increases rapidly. Numerical solution on a PC becomes time consuming; therefore, an analytical approach is desirable to obtain the pulse-modulated spectrum for modulations that can be adequately represented by analytic functions.

2.2 Calculated Results

Fortunately, the modulations of interest have a frequency content that is significantly lower than the carrier frequency. That is, the modulation waveform $p_X(t)$ has a low-frequency spectrum $P_X(\omega)$, where $P_X(\omega) \sim 0$ for $|\omega| < \omega_{max}$ and $\omega_{max} << \omega_c$. This implies that $f(t)$ is an analytic function of infinite duration, but even when it is truncated we still find that $\omega_{max} << \omega_c$ with $P_X(\omega_{max}) << P_X(\omega_c)$, and the spectrum is approximately band-limited. The positive frequency components of the modulated signal are

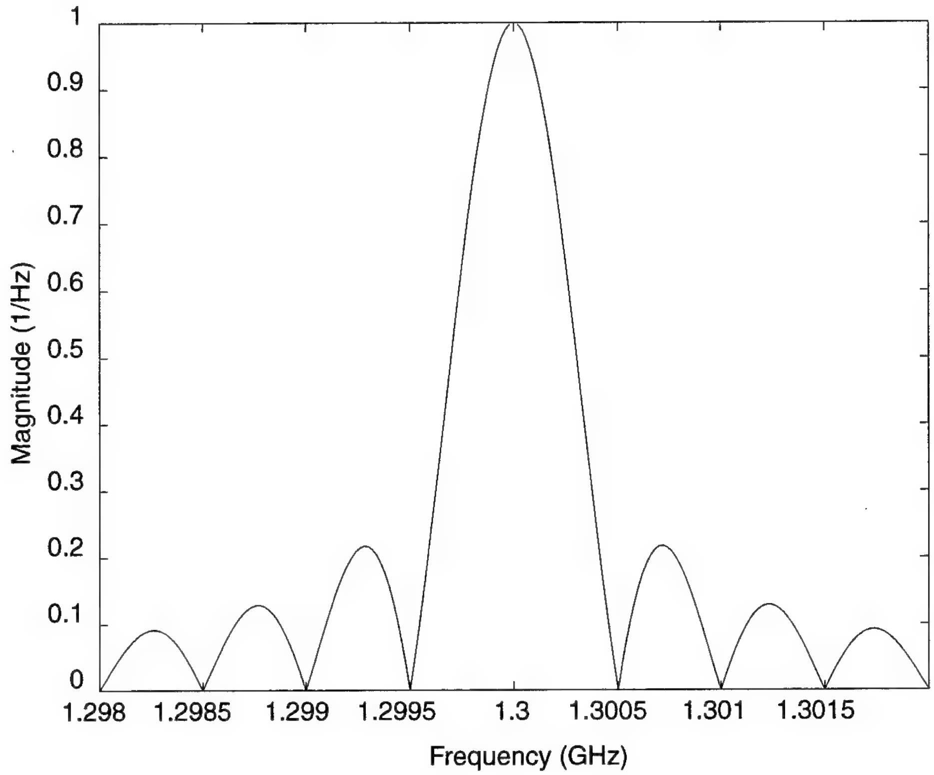
$$F_0\left(\omega^+\right) = \frac{P_X\left(\omega-\omega_c\right)}{2T_0} = \frac{\sin\left(X\left(\omega-\omega_c\right)\right)}{T_0\left(\omega-\omega_c\right)} = A_c(\omega)e^{j\theta_c(\omega)}, \tag{11}$$

with peak magnitude $\frac{1}{2}D_{rf}$. So we can calculate $\frac{1}{2}P_X(w)$ with the appropriate normalization, and shift the spectrum to be centered on f_c to approximate the positive frequency continuous spectrum $F_0(\omega^+)$.[2] Using the previous example modulation, $\frac{1}{2}P_X(\omega)$ is shown in figure 3 without amplitude correction, so the peak magnitude is $X = T/2$. As can be seen from figure 2, the correct amplitude is obtained by normalization to the time-average or dividing by $t_{max} = T_0$. That is, the analytical result is normalized to have the correct rf duty factor in one complete period of the modulation waveform to correspond to the FFT result. The complete spectrum $F(\omega)$ would have impulses at the PRF, but the fundamental spectrum $F_0(\omega)$ is sufficient to characterize the spectral magnitude and FBW of the modulated signal. A convolution routine could also be used to calculate $F_0(\omega) = P_X(\omega) \int C(\omega)/4\pi T_0$ from the modulation and carrier spectra. In this manner, more complex carrier behavior could be included, but a single carrier frequency is sufficient in our analysis.

Thus, the spectrum for the infinite-duration pulse-modulated rf signal can be readily approximated by FFT or direct calculation. For ideal rectangular pulse modulations, the transform is analytic but, in general, the modulation pulse is more complicated with only a digitized waveform representation. We have shown that for low-frequency modulations compared to a single carrier, the carrier frequency need not be resolved in the sampled transient. Then measurement of one period of the repetitive modulation waveform is sufficient to characterize the periodic signal. Then one can use numerical transform or analytical techniques to obtain the fundamental signal spectrum, and we must realize that this represents

[2] A. V. Oppenheim, Willsky, A. S., and Nawab, *S. H., Signals and Systems*, 2nd ed.

Figure 3. Calculated spectrum of a pulsed rf signal at 1.3 GHz.



the envelope of the impulses contained in the actual spectrum. Developing such tools is a first step in analyzing the spectrum as a function of the modulation parameters. Once obtained, the spectral content is useful in estimating the propagation, coupling, and scattering of modulated rf signals.

3. Additional Modulation

Now consider an additional rectangular pulse AM with pulsewidth T_1 and AM PRF = $1/T_2$. The composite signal is then $q(t) = g(t)f(t) = g(t)p_X(t)c(t)$, where $g(t) = p_X(t)$ is periodic with a period $T_2 \sim kT_0$ for integer k, and $Y = T_1/2$. Since both $g(t)$ and $f(t)$ are even functions of time, so is $q(t)$, with fundamental periods T_0 and $T_2 \sim kT_0$. The Fourier series coefficients for $p_Y(t)$ are $\beta_n = P_Y(n\omega_2)/T_2$, where $\omega_2 = 2\pi/T_2$. The partial sum is

$$q_N(t) = \sum_{n=-N}^{N} \sum_{m=-M}^{M} \alpha_m \beta_{n-m} e^{jn\omega_2 t} = \sum_{n=-N}^{N} \gamma_n e^{jn\omega_2 t} , \tag{12}$$

where the sum over $2M + 1$ terms is a convolution sum for the $2N + 1$ coefficients γ_n. Then

$$Q_0(n\omega_2) = \gamma_n T_2 = T_2 \sum_{m=-M}^{m=+M} \alpha_n \beta_{n-m} , \tag{13}$$

but this convolution is also numerically intensive. The convolution is equivalent to scaling the spectral amplitude in equation (11) because the additional modulation changes the limits of integration in equation (4) to correspond to the fundamental period of the repetitive modulation waveform. Thus the Fourier series coefficients for $q(t)$ are weighted by the new duty factor and

$$Q_0(\omega) = \frac{T_1\, A_c(\omega)}{T_2}\, e^{j\theta_c(\omega)} = D_{AM}\, F_0(\omega) \tag{14}$$

is the Fourier transform of the fundamental signal $q_0(t)$. The spectrum for the infinite-duration signal $Q(\omega)$ is a sequence of impulses at $1/T_2$ (and $1/T_0$) with the envelope defined by equation (14). For idealized modulation waveforms, the spectrum $P_X(\omega)$ can be obtained, normalized to the correct time-average, and scaled by one-half to account for the modulated carrier.

Consider a numerical approximation where the carrier frequency is not resolved and the FFT is used to obtain the spectrum of sampled transients. Here we use at least 21 samples in the smallest pulse and linear spacing of the transient data for one complete period of the modulation waveform. The rf carrier is 1.3 GHz, $T = 2$ µs, with rf PRF = 20 kHz, and we apply an additional modulation characterized by the AM duty factor $D_{AM} = T_1/T_2$ and AM PRF = $1/T_2$. For example, let $T_1 = 0.5$ ms and $T_2 = 2$ ms, so that the AM repetition frequency is 500 Hz and $D_{AM} = 0.25$. The modulation waveform contains 10 pulses as shown in figure 4(a) with the FFT in figure 4(b). The FFT result has been scaled by one-half to account for the modulated carrier and frequency-shifted to correspond to $|Q_0(\omega)|$ for the modulated rf signal. In figure 4(c), I show an 80-kHz frequency span with dominant impulses at the PRF = 20 kHz, since now the FFT time window includes repetitive pulses. The FBW of the spectral envelope depends on T (see fig. 4(b)) but the FBW of the impulses depends on T_1, as can be seen in figure 4(c). The impulses appear at the PRF = 20 kHz and at $1/T_1 = 2$ kHz with FBW = $2/T_1 = 4$ kHz. Reducing T_1 (while

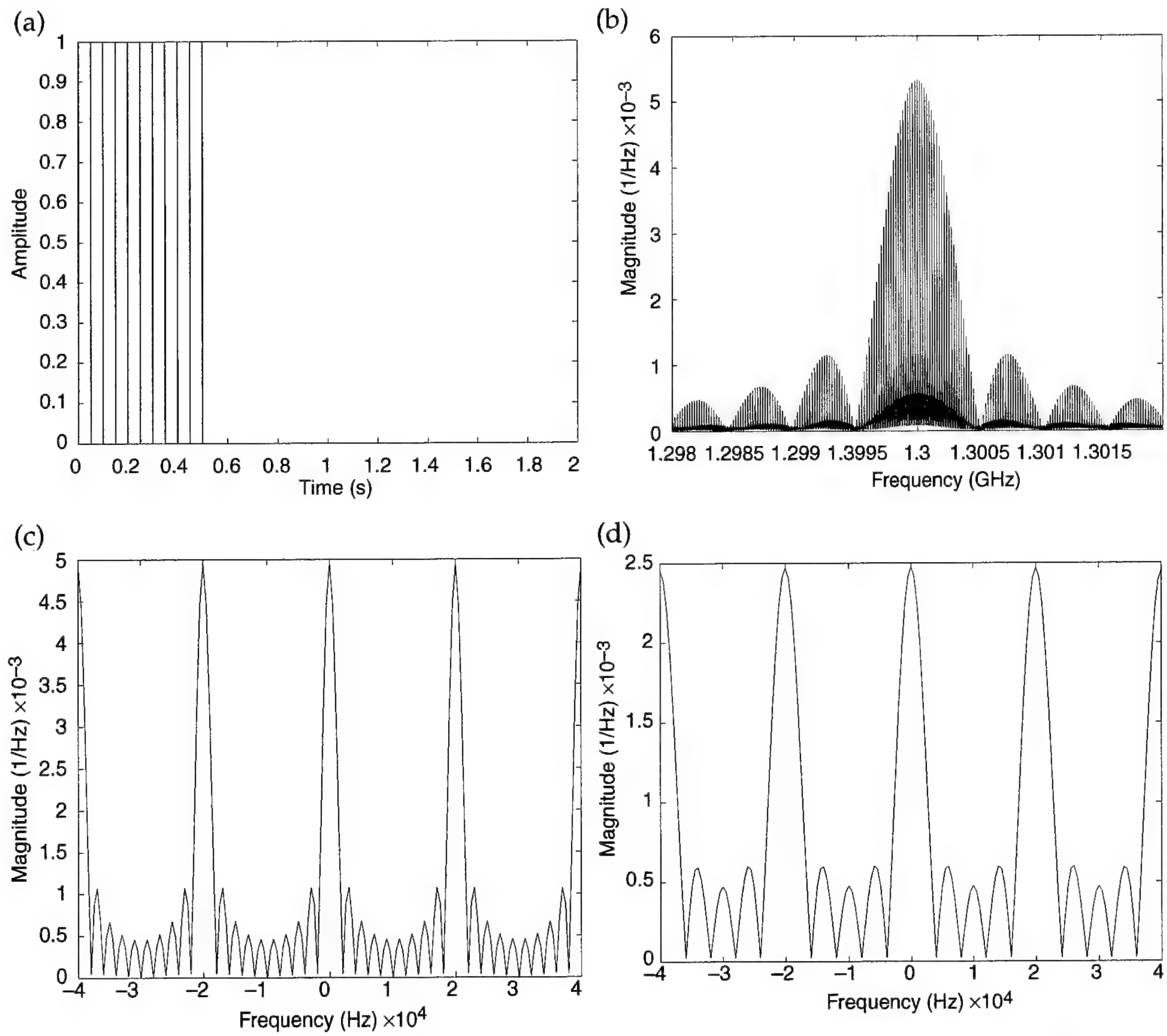


Figure 4. (a) Single period of a modulation waveform with 20 pulses, (b) frequency-shifted FFT (times one-half) of fundamental modulation, (c) FFT (times one-half) of fundamental modulation showing spectral line characteristics, and (d) FFT (times one-half) of fundamental modulation for T_1 = 0.25 ms.

holding the other parameters fixed) results in fewer impulses with a larger FBW, as shown in figure 4(d), where now $T_1 = 0.25$ ms and $D_{AM} = 0.125$. Conversely, increasing T_1 results in more impulses with lower FBW and the spectral amplitude is proportional to D_{AM}. Although not shown, this is exactly what is observed on a spectrum analyzer.

The same result can be obtained by the appropriate normalization of the single pulse modulation spectrum calculated analytically or by FFT of the modulation waveform. That is, we need only the pulse shape and the modulation duty factors to generate the pulse-modulated power spectrum. The frequency-shifted FFT for $p_X(t)$ sampled in a 2-ms time window is presented in figure 5. The result is scaled for 10 pulses to have the correct time-average as in figure 4(a), and divided by 2 to account for the sinusoidal carrier for comparison to figure 4(b). The peak amplitude $\frac{1}{2}D_{rf}D_{AM} = 0.005$ and the spectral envelope FBW = $2/T$ are roughly the

same as in figure 4(b), with the difference owing to the finite rise- and fall-times of the rectangular pulse modulation. The fundamental spectrum is continuous, but since the modulation is periodic, the complete spectrum $Q(\omega)$ would contain sharp impulses at the AM PRF ($1/T_2 = 500$ Hz) and at the rf PRF ($1/T_0 = 20$ kHz). Alternatively, we could calculate the spectrum of $Q_0(\omega)$ with the appropriate normalization and frequency shift. The calculated spectral envelope $P_X(\omega - \omega_c)T_1/(2T_0T_2)$ is shown in figure 6 and represents the average power, where now there is no reduction in amplitude due to nonideal rectangular pulses. The calculation of analytic functions can be more accurate than a numerical approach, but in both cases, care must be taken to avoid resolution errors due to poor sampling.

Figure 5. Single-pulse FFT (times one-half) in 2-ms time window scaled for 10 pulses.

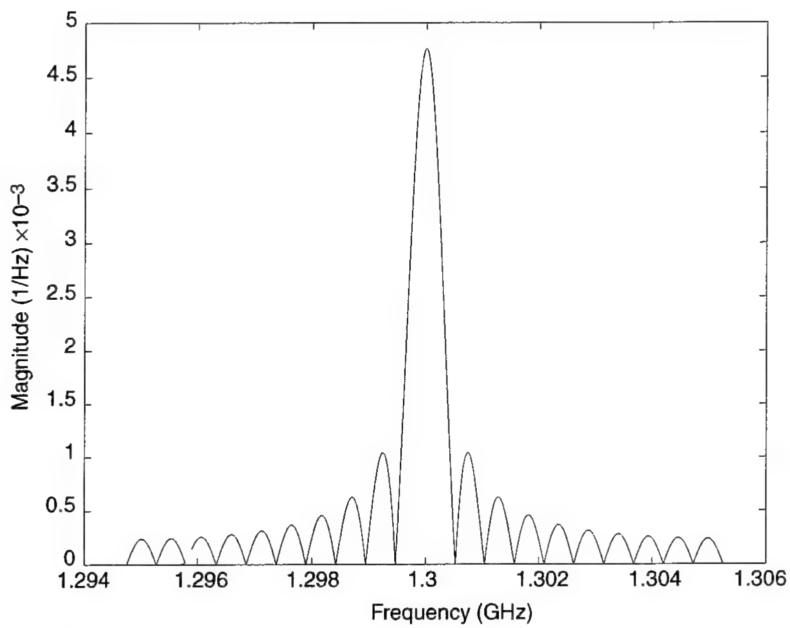
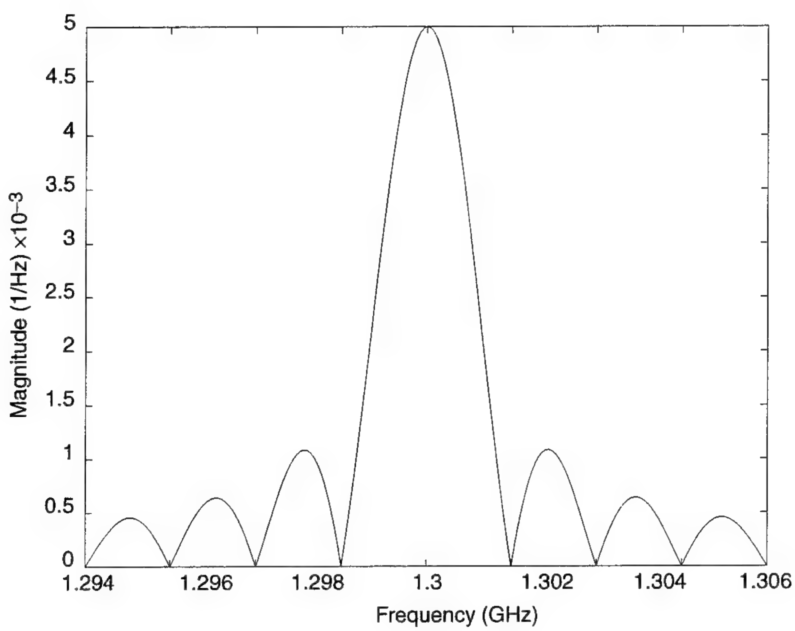


Figure 6. Calculated fundamental spectrum scaled by duty factor.



4. Modulated rf Bursts

Since only a finite number of pulses are transmitted, the rf signal is actually a transient burst $b(t)$ of a periodic waveform. For a whole number of modulation periods, the duration does not change the modulation duty factor since the signal is repetitive, but it is needed to calculate the average energy transmitted. We show this by windowing the periodic waveform with a single rectangular pulse $p_Z(t)$ to truncate the pulse train at the maximum time t_{max}; so let $Z = t_{max}/2$.[1] The spectrum $P_Z(\omega)$ obtained by the FFT has unit magnitude, since the time-average for this window function is unity. The fundamental spectrum can be represented by a convolution with the normalized $P_Z(\omega)$, $B_0(\omega) = P_Z(\omega) \otimes Q_0(\omega)/2\pi$, or

$$B_0(\omega) = \frac{\sin(\omega Z)}{4\pi\omega Z} \otimes Q_0(\omega) \ . \tag{15}$$

Thus, we could calculate the rf burst spectrum as a convolution of analytical or FFT results and shift the result to ω_c, but for measured data, FFT techniques are often preferred for numerical efficiency. The modulation waveform is shown in figure 7(a), truncated to $t_{max} = 10$ ms, which results in 50 rf pulses (or 5 AM pulses) with unit peak power transmitted. The frequency-shifted FFT result (scaled by one-half) for this example is shown in figure 7(b), and this is the power spectrum measured on a spectrum analyzer. The FFT result does not depend on the duration when the modulation duty factors are not modified (i.e., a whole number of modulation periods transmitted). In figure 7(c), we show an 80-kHz frequency span for comparison to figure 4(c). The impulses in figure 4(c) represent another envelope of sharp impulses at the lowest PRF (500 Hz in this example) as shown in figure 7(d).

The FFT results for $q_0(t)/2$ and $p_z(t)$ are shown in figures 8(a) and 8(b), respectively. The spectral envelope depends primarily on T with fine features related to T_1, while the number of impulses depends on T_2. Truncation of the transient to an rf burst signal does not affect the FFT result or the power spectrum as measured on a spectrum analyzer. However, the total average energy transmitted is proportional to the duration (or dwell time) so that the average energy is obtained by scaling the power spectrum by t_{max}. The frequency-shifted energy spectrum for a 50-pulse burst is shown in figure 9, where the amplitude is based on a peak transmitted power of 30 dBm (1 W). When only the spectral envelope $B_0(\omega)$ is of interest, it can be calculated directly (with the appropriate normalization). The calculated energy is shown in figure 10 and represents the average energy spectral envelope. The energy spectrum is still a sequence of impulses at the modulation repetition frequencies as shown in figure 9, with magnitude defined by this envelope.

[1]A. Papoulis, *The Fourier Integral and Its Applications*.

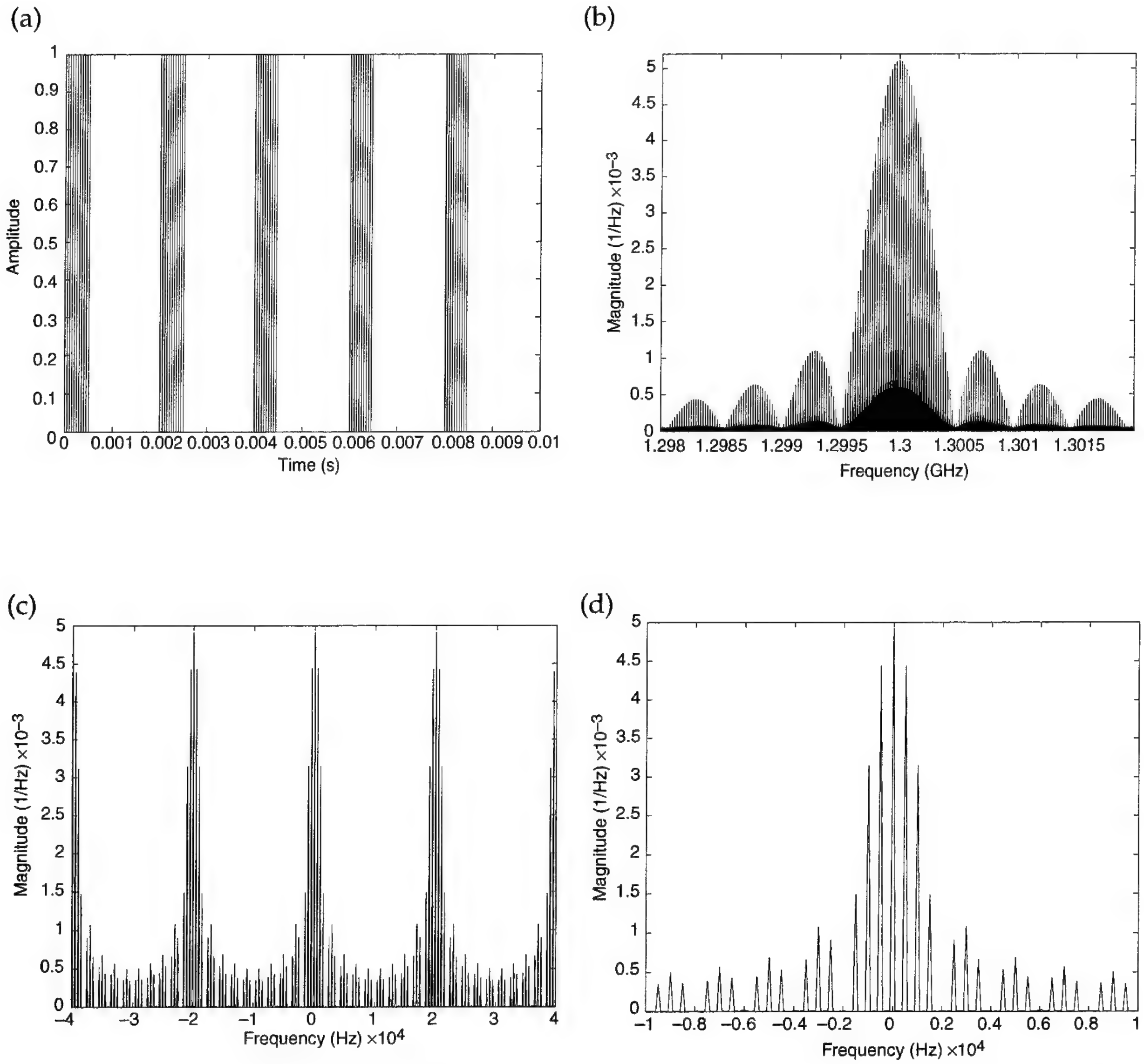


Figure 7. (a) Modulation waveform for 50-pulse rf burst, (b) scaled FFT result shifted to modulated carrier, (c) FFT result (times one-half) over 80-kHz span for comparison to figure 4(c), and (d) FFT result (times one-half) over 20-kHz span to show spectral line characteristics.

Figure 8. (a) FFT result (times one-half) for two periods of modulation waveform and (b) FFT result for truncation time window.

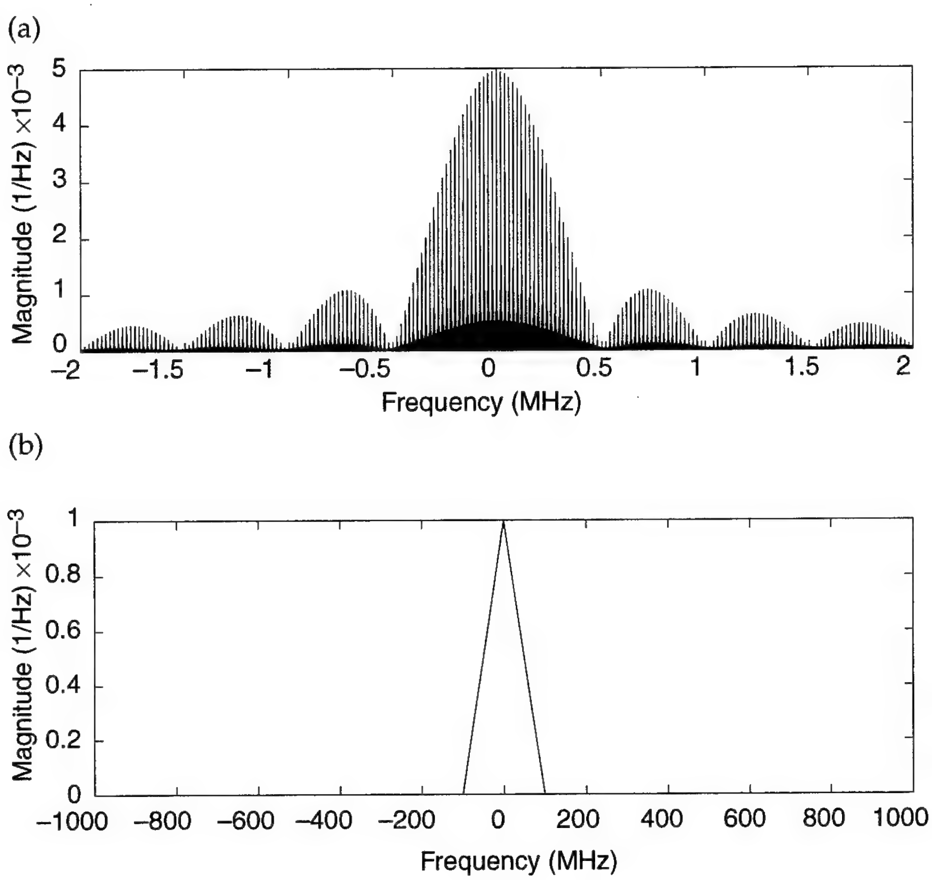


Figure 9. Energy spectrum by convolution of FFT results.

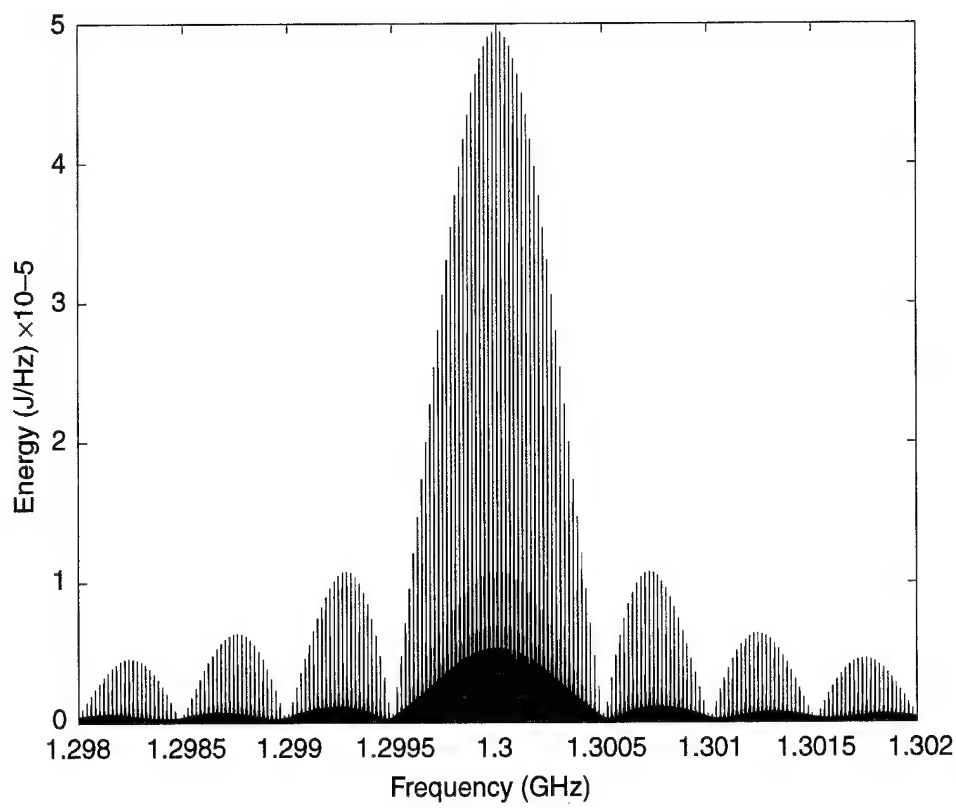
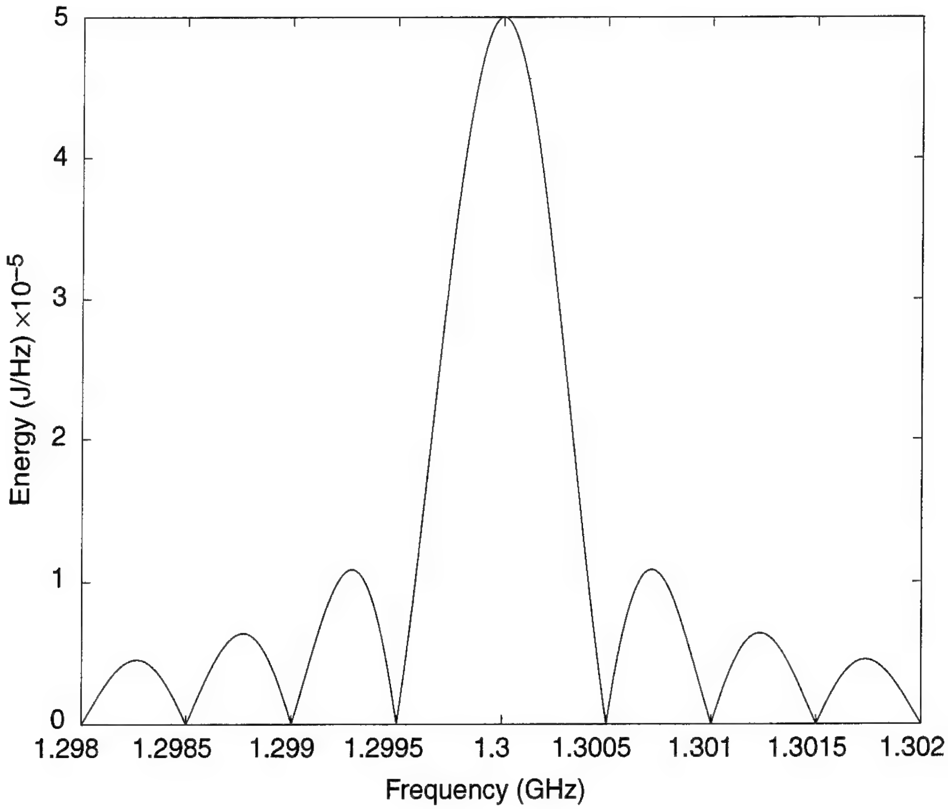


Figure 10. Calculated energy spectrum for 10-ms rf burst signal.



5. Discussion

I have shown that the spectrum of pulse-modulated signals can be readily estimated when the modulation waveform has a low-frequency content compared to the carrier frequency. Given a single period of the modulation waveform, the fundamental spectrum can be obtained by FFT for digitized data or by direct calculation for idealized modulations. In either case, the result is the spectral envelope of the power spectrum that contains impulses at all the modulation repetition frequencies. These impulses can be calculated according to the Fourier series representation or obtained by FFT of several periods of the modulation waveform. The truncated or rf burst signal spectrum has the same average power spectrum with average energy proportional to the burst duration. The rf burst power spectrum is obtained by normalization to the modulation total duty factor, with the average energy obtained by scaling for the burst duration.

The modulation pulse for the rf carrier is typically a nonideal rectangular pulse with rise- and fall-times of up to 10 percent of the full width and an amplitude with variations of ±10 percent. The AM modulation waveforms, which determine the signal total duty factor, have negligible rise- and fall-times, so they can be considered ideal pulses. Our approach then is to obtain the FFT for the digitized modulation pulse and convolve this power spectrum with the calculated spectrum for the AM waveform. This is equivalent to normalization by the AM duty factor so that only the rf pulse modulation waveform is required to calculate the signal power spectrum. The effect of a finite rise- and fall-time is reflected in the FFT results shown here since the zero-to-peak rise- and fall-times are $dt \sim$ 10 ns. The difference between the numerical and analytical results (compare fig. 5 and 6) is associated with the difference in the time-average owing to different pulse shapes. The error in the peak magnitude is negligible for rise- and fall-times of up to about 100 ns in this example.

To summarize, the rf modulating pulsewidth determines the FBW of the spectral envelope about the carrier frequency, with dominant impulses at the PRF. An additional AM introduces more impulses with a secondary envelope that depends on the AM pulsewidth (T_1). The spectral magnitude is reduced according to the modulation duty factor D_{AM}, but the overall FBW is unchanged. The introduction of another AM reduces the peak magnitude by D_{AM} but increases the number of impulses in the spectrum. This is shown in table 1 for some typical rf and AM modulation parameters. For peak transmitted power $P_0 = 30$ dBm, the spectral magnitude represents the average power $P_{avg} = \frac{1}{2} D_{rf} D_{AM} P_0$. For an rf burst signal, the average energy is determined from the duration $E_{avg} = P_{avg} t_{max}$.

Thus, if one desires a spectral envelope that has a narrow FBW, then a long pulse should be used for the rf modulation. A broader FBW is obtained for a shorter pulsewidth with a corresponding decrease in the spectral magnitude according to D_{rf}. The spectrum is a sequence of sharp impulses whose number and amplitude depend on the AM modulation.

Table 1. Spectral characteristics of an rf burst of duration t_{max} and 1-W peak transmitted power.

rf modulation parameters (μs)			AM parameters (ms)		FWB (kHz)	Impulse frequency (kHz)	P_{avg} (mW)	E_{avg} (nJ)
T	T_0	t_{max}	T_1	T_2				
2	1000	500	—	—	1000	1	1	500
2	200	100	—	—	1000	5	5	500
2	50	25	—	—	1000	20	20	500
2	50	100	0.25	1	1000	1	5	500
2	50	100	0.5	2	1000	0.5	5	500
10	2000	200	—	—	200	0.5	2.5	500
10	2000	400	2	4	200	0.25	1.25	500
10	1000	200	2	4	200	0.25	2.5	500
10	200	80	0.5	2	200	0.5	6.25	500
30	2000	67	—	—	67	0.5	7.5	500
30	2000	133	2	4	67	0.25	3.75	500
—	—	200	0.05	10	40	0.1	2.5	500

However, the overall power spectrum envelope is determined by the rf modulation with peak magnitude given by the time-average of the rf signal. Given a fixed D_{rf}, the total average power with AM is reduced by D_{AM} unless P_0 is increased to maintain the same time-average. Alternatively, for fixed transmitted power, D_{rf} must be increased to compensate for D_{AM} to obtain the same average power in the transmitted signal. In terms of average energy, the duration is a parameter, so different combinations of rf and AM modulations could have equivalent E_{avg} in the transmitted signal as shown in table 1. The rf pulsewidth and the lowest modulation repetition frequency are the important parameters for the spectral content of the transmitted signal, while the modulation duty factors control the spectral amplitude. The rf and AM parameters can be appropriately adjusted to obtain a desired power spectrum with the energy spectrum determined by the duration of the rf burst signal.

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13. ABSTRACT *(Maximum 200 words)*

The parameters that characterize a rectangular-shaped pulse-modulated sinusoidal signal are the carrier frequency, the pulsewidth, the repetition frequency, and the number of pulses in or the duration of the signal. We use a Fourier series representation to show the influence of these parameters on the spectrum of a pulse-modulated signal at a microwave carrier frequency. When an additional amplitude modulation is applied at audio frequencies, the resulting transient cannot be efficiently analyzed with numerical transform techniques. We present approximate numerical and analytical techniques to obtain the frequency spectrum of such signals. This approach allows the near-real-time spectral analysis of modulated signals. Thus, the resulting spectrum can be easily calculated for idealized modulation waveforms. A typical example is presented and the effect of pulse modulation on the spectral content of an rf signal burst is discussed.

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